## Quadratic Equations - Part 1

**Definition:**

A quadratic equation is a polynomial equation of the second degree, meaning it contains at least one term that is squared. The general form of a quadratic equation is:

$$ax^2+bx+c=0$$

where $a$, $b$, and $c$ are constants with $a \neq 0$, and $x$ is the variable.

**Solutions:**

1. **Roots**: The solutions to a quadratic equation are called its roots, or zeros. A quadratic equation can have zero, one, or two real roots.
2. 
3. **Discriminant**: The discriminant ($\Delta$) of a quadratic equation is given by the expression $b^2-4ac$. It determines the nature of the roots:
4. 
   - If $\Delta>0$, the equation has two distinct real roots.
   - If $\Delta=0$, the equation has one real root (a repeated root).
   - If $\Delta<0$, the equation has two complex (non-real) roots.

**Methods of Solving:**

1. **Factoring**: If a quadratic equation can be factored into two binomials, setting each binomial equal to zero allows for finding the roots.
2. 
   - Example: $x^2+5x+6=0$ can be factored as $(x+2)(x+3)=0$, yielding roots $x=-2$ and $x=-3$.
3. **Quadratic Formula**: The quadratic formula is a general method for finding the roots of any quadratic equation. It is given by:
4. 
   $$x=\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
5. 
   - This formula applies regardless of whether the roots are real or complex.

6. **Completing the Square**: By completing the square, a quadratic equation can be transformed into a perfect square trinomial, from which the roots can be easily determined.

7.

**Example:**

**Solve the quadratic equation $2x^2-5x+2=0$.**

1. **Using Quadratic Formula**:

2.
   - Identify $a=2$, $b=-5$, and $c=2$.
   - Apply the quadratic formula:

     $$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(2)}}{2(2)} \quad =\frac{5\pm\sqrt{25-16}}{4}$$

     $$=\frac{5\pm\sqrt{9}}{4} \quad =\frac{5\pm 3}{4}$$
   - This yields two roots: $x_1=\frac{8}{4}=2$ and $x_2=\frac{2}{4}=\frac{1}{2}$.

3. **Checking with Discriminant**:

4.
   - Calculate the discriminant: $\Delta=(-5)^2-4(2)(2)=25-16=9$.
   - Since $\Delta>0$, the equation has two distinct real roots, as expected.

**Conclusion:**

Quadratic equations are fundamental in algebra and have wide-ranging applications in various fields of science, engineering, and mathematics. Understanding methods for solving quadratic equations, such as factoring, the quadratic formula, and completing the square, allows for the efficient determination of roots and the analysis of quadratic relationships.