Highest Common Factor (HCF) - Part 1

Definition: The Highest Common Factor (HCF), also known as the Greatest Common Divisor (GCD), of two or more numbers is the largest positive integer that divides each of the numbers without leaving a remainder.

Finding HCF:

1. **Prime Factorization Method**:
2.
   - Find the prime factors of each number.
   - Identify the common prime factors.
   - Multiply these common prime factors to find the HCF.

   Example: Find the HCF of 24 and 36:
   - Prime factors of 24: $23 \times 32^3 \times 3$
   - Prime factors of 36: $22 \times 322^2 \times 3^2$
   - Common prime factors: 222^2 and 33
   - HCF = $22 \times 3 = 122^2 \times 3 = 12$
3. **Division Method (Euclidean Algorithm)**:
4.
   - Divide the larger number by the smaller number.
   - Replace the larger number with the remainder.
   - Repeat the process until the remainder is zero.
   - The divisor at this step is the HCF.

   Example: Find the HCF of 48 and 60:
   - $60 = 48 \times 1 + 1260 = 48 \times 1 + 12$
   - $48 = 12 \times 4 + 048 = 12 \times 4 + 0$
   - HCF = 12

Properties of HCF:

1. **HCF is Divisor**: The HCF of two or more numbers divides each of the numbers without leaving a remainder.
2.
   - Example: HCF of 24 and 36 (HCF = 12) divides both 24 and 36.
3. **HCF is Unique**: For any given set of numbers, the HCF is unique, regardless of the method used to find it.
4.
5. **Relation with LCM**: The product of the HCF and the Least Common Multiple (LCM) of two numbers is equal to the product of the numbers themselves.
6.

- Mathematically, for two numbers a and b:

  HCF(,)×LCM(,)= × HCF(∠, /)×LCM(∠, /)=∠× / .

Applications of HCF:

1. **Simplifying Fractions**: HCF is used to simplify fractions by dividing both the numerator and the denominator by their HCF.
2.
   - Example: $\frac{24}{36}$ can be simplified to $\frac{2}{3}$ by dividing both numerator and denominator by their HCF (12).
3. **Comparing Ratios**: HCF is used to compare ratios by finding the HCF of the terms in the ratio.
4.
   - Example: If the ratio of two numbers is 12:18, the simplified ratio would be 2:3 after dividing both terms by their HCF.
5. **Solving Problems in Number Theory and Algebra**: HCF plays a fundamental role in various mathematical problems, including those related to divisibility, factors, and multiples.
6.

Conclusion:

The Highest Common Factor (HCF) is a fundamental concept in mathematics used to find the largest common divisor of two or more numbers. It can be calculated using methods such as prime factorization or the division method (Euclidean algorithm). Understanding the properties and applications of HCF is essential for simplifying fractions, comparing ratios, and solving various mathematical problems across different branches of mathematics.