

## Average - Part 2

### Weighted Average:

1.

**Definition:** In a weighted average, each value in the set is multiplied by a corresponding weight, reflecting its importance or contribution to the overall average.

2.

- For a set of  $\emptyset$  values  $x_1, x_2, \dots, x_{\emptyset}$  with corresponding weights  $w_1, w_2, \dots, w_{\emptyset}$ , the weighted average  $\bar{x}$  is calculated as:

$$\bar{x} = \frac{x_1 \cdot w_1 + x_2 \cdot w_2 + \dots + x_{\emptyset} \cdot w_{\emptyset}}{w_1 + w_2 + \dots + w_{\emptyset}} \quad \bar{x} = \frac{x_1 w_1 + x_2 w_2 + \dots + x_{\emptyset} w_{\emptyset}}{w_1 + w_2 + \dots + w_{\emptyset}}$$

3.

### Applications:

4.

- Weighted averages are used when certain values have more significance or influence than others.
- Examples include calculating course grades based on assignments' weights, financial indicators like weighted average cost of capital (WACC), and composite indices in economics.

### Harmonic Mean:

1.

**Definition:** The harmonic mean is a type of average calculated by taking the reciprocal of the arithmetic mean of the reciprocals of the values in the set.

2.

- For a set of  $\emptyset$  values  $x_1, x_2, \dots, x_{\emptyset}$ , the harmonic mean  $H$  is calculated as:

$$H = \frac{\emptyset}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_{\emptyset}}} \quad H = \frac{\emptyset}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_{\emptyset}}}$$

3.

### Applications:

4.

- The harmonic mean is used in scenarios where rates or ratios are involved, such as calculating average speeds, average rates of return, or average prices per unit.

### Geometric Mean:

1.

**Definition:** The geometric mean is a type of average calculated by taking the  $\sqrt[n]{}$ th root of the product of  $n$  values in the set.

2.

- For a set of  $n$  values  $x_1, x_2, \dots, x_n$ , the geometric mean  $G$  is calculated as:

$$G = \sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}$$

3.

### Applications:

4.

- The geometric mean is used when dealing with growth rates, ratios, or exponential growth, such as calculating average compound annual growth rates (CAGR) or average rates of change.

### Relationship between Averages:

1.

**Harmonic Mean and Arithmetic Mean:** For a set of positive numbers, the harmonic mean is always less than or equal to the arithmetic mean.

2.

3.

**Geometric Mean and Arithmetic Mean:** For a set of positive numbers, the geometric mean is always less than or equal to the arithmetic mean, with equality holding when all values are equal.

4.

### Conclusion:

Understanding different types of averages, including weighted averages, harmonic mean, and geometric mean, provides a more nuanced perspective on analyzing data and calculating averages in various contexts. Each type of average has its own applications and is used based on the nature of the data and the specific requirements of the analysis. By utilizing appropriate averages, mathematicians, statisticians, and analysts can derive more accurate insights and make informed decisions in their respective fields.